is an antiderivative of g on $(0,1]$. Thus, since F is continuous on $[0,1]$ (and g is integrable of $[0,1]$), we have by the Fundamental Theorem of Calculus (for $x \in (0,1)]$ that $\int_0^x g(t)\,dt = F(x) - F(0) = F(x)$ and the equation holds trivially for $x = 0$. Putting $x_n = e^{-n\pi}$, $n \in \mathbb{N}$, we find that although $\lim_{n\to\infty} x_n = 0$,

$$\lim_{n\to\infty} \frac{\int_0^{x_n} g(t)\,dt}{x_n} = \lim_{n\to\infty} \frac{\sin(-n\pi) - \cos(-n\pi)}{2} = \lim_{n\to\infty} -\frac{\cos(n\pi)}{2} = \lim_{n\to\infty} \frac{(-1)^{n+1}}{2}$$

which does not exist and, hence, $\lim_{x\to 0^+} \int_0^x g(t)\,dt/x$ does not exist.

In comparing f with g, notice that the intervals between consecutive zeros of g are exponentially smaller than those for f. For f, the length of the interval

$$\left[\frac{1}{(n+1)\pi}, \frac{1}{n\pi}\right]$$

is $l_n(f) = 1/n(n+1)\pi$, while for g, the length of the interval $[1/e^{(n+1)\pi}, 1/e^{n\pi}]$ is $l_n(g) = (e^{\pi} - 1)/e^{(n+1)\pi}$ and we observe that $\lim_{n\to\infty} l_n(g)/l_n(f) = 0$. Thus the more highly oscillatory nature of g appears to prevent the right differentiability of its integral function at the origin.

Finally, in light of these (and other) examples, the conjecture can be made (although the author is by no means certain of its validity) that if in addition to conditions (a)–(c), the zeros of f form a null sequence of distinct terms whose associated sequence of distances between consecutive zeros is asymptotically proportional to $1/n^{\alpha}$ for some $\alpha > 1$, then $F'_{+}(0) = 0$.

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Geographical Boundary Extrema

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H. Thurston [**1**] has given Florida as an example of a geographical entity that has its highest elevation on the boundary. However, upon checking a detailed map [**2**] one finds, contrary to the claim, that the highest point in Florida is not on the border with Alabama, but rather is located approximately half a mile south of the border (just south of the village of Lakewood, Walton County; T. 6N, R. 20W, section 30). In [**3**, p. 4] the maximum elevation is given as 345 feet whereas the values at the border appear from the map to be bounded above by 340 feet.

For a valid example consider the province of Alberta. Since the B.C.–Alberta border in the vicinity of 12,294-foot Mt. Columbia is determined by the watershed line [**4**], and since Alberta has no interior points, or other boundary points, that high, Mt. Columbia's summit is a boundary maximum for Alberta.

British Columbia itself presents an interesting example that emphasizes the importance of the domain of definition in extremal problems. Following the decisions of the 1903 Alaska Boundary Tribunal the International Boundary Commission published a report [**5**] dealing with the survey and demarcation of the boundary between B.C. and Alaska. Since the placing of a permanent marker on the peak of Mt. Fairweather (approximate height 15,230 feet) was not feasible, the report [**5**, p. 175, boundary point 164] states only: "Station mark: the highest part of the snow cap as it was in 1907." This statement provides the present legal description of the fixed boundary point. Coordinates of this snow cap had been determined from surveys, but since even for the key boundary point at Mt. St. Elias the calculations of two surveying parties differ by about 200 feet [**5**, p. 134], the actual location of the boundary point at Mt. Fairweather must be considered, in surveying terms, to be only very approximately known.

Because snow accumulation since 1907 has surely changed the location of the highest part of the snow cap, the station mark no longer exists in a physical sense. Further, because of the inaccurate survey, we cannot state from a practical viewpoint—unless the drift has been very large—whether the present summit of Mt. Fairweather is legally in B.C. or in Alaska. In the former case we would have an interior absolute maximum for B.C.; in the latter case the present legal absolute maximum for B.C. is somewhere on Fairweather's slopes. It should be noted that because of the watershed definition, this situation cannot occur on the B.C.–Alberta border and indeed the border has been recently redrawn after it was noticed that a body of water was flowing the "wrong way!"

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