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Proportions in the Architecture Curriculum

Roger Herz-Fischler presents a revised version of a chapter entitled "Proportions" that appeared in the problems part of his book, *Space, Shape and Form /An Algorithmic Approach*, developed for a mathematics course he taught in the School of Architecture at Carleton University from 1973-1984.

Example of reckoning of a pyramid,
Height 250, base 360 cubits.
What is its seked*?......(Answer: 5+1/25 hands)*.
From the Rhind Papyrus[1]

Preface

Starting in 1973 and ending in 1984, I developed material for a mathematics course in the School of Architecture at Carleton University. I experimented with different topics and eventually wrote a book for my students entitled *Space, Shape and Form /An Algorithmic Approach.*[2] The material in this paper is a revised version of a chapter entitled "Proportions" that appeared in the problems part of the book.

So that the reader can better understand the raison d'être of what follows, and also to decide if they agree with my views on the teaching of mathematics in the architectural curriculum, I shall first describe how the chapter, and the rest of the book, came to be written.

In 1972 I was asked to teach the mathematics course for first-year students of architecture. Since I was essentially free to choose the topics for the course I decided to introduce some material dealing with the use of mathematical proportions in architecture. After a few years I discovered that much of what I had read in the literature was at best not completely evident and at the worst out and out nonsense. This in turn led to an apology to my students and a change in my own research from theoretical probability into the fields of the history of mathematics, proportions — in particular the "golden number" — and what I refer to as the sociology of mathematical myths.[3] Indeed parts of what I present below are based on my own research.

On the other hand the course itself came more and more oriented to what we might call applications of geometry to architecture. Together with this change in approach, I developed a book containing theory in the first part and a large collection set in the second part.[4] I felt that analysing some historical examples of proportions in the first problem chapter would not only be a good mathematical "warm-up" for the more complicated problem chapters, but would also be of particular interest to the students. As a secondary agenda, I hoped to put the students on guard against many of the dubious statements concerning proportions that appeared in the architecture literature.

The problem of proportions

In my book *The Shape of the Great Pyramid* [Herz-Fischler 2000] I have devoted a chapter to the philosophical and practical problems involved in trying to reconstruct the design proportions of an architectural structure. For the purpose of introducing proportions to students of architecture I found that the following two well-known examples were not only of intrinsic interest to the students, but also made them realize how important it was to treat these matters with great care.

The first example—this is explored further in Case Study 1—involves the Great Pyramid. As indicated by the example cited in the opening quotation, we know that the ancient Egyptians had a well-defined theoretical method of indicating the slope of a pyramid. Now consider the drawing of the excavated base of the mastaba at Medum, Figure 1.The horizontal lines are well defined as being one cubit apart, but there is not the slightest mark to indicate how the slope was determined. In view of the ancient Egyptian mathematical texts one would suppose that the *seked* approach was used. However a detailed examination of the archaeological literature suggests that it far from being certain.

The second example—explored further in Case Study 10— involves Le Corbusier's 1927 villa at Garches. What graduating architecture student—not to mention the numerous books and articles which use it to explain Le Corbusier's infatuation with the golden number—has not seen the view in Figure 2 that was published in Le Corbusier's *Oeuvre complète*? However despite the boldface formula proclaiming the use of the golden number at Garches, this drawing is what one might call a "plant", for, as I discovered in the course of my research, not only was it drawn at least eighteen months after the building was completed, but the original sketches showed that Le Corbusier used his "place of the right angle" to design the villa. Further Le Corbusier's own writing show that he had previously been extremely critical of the use of the golden number in design [Fischler 1979; Herz-Fischler 1984].

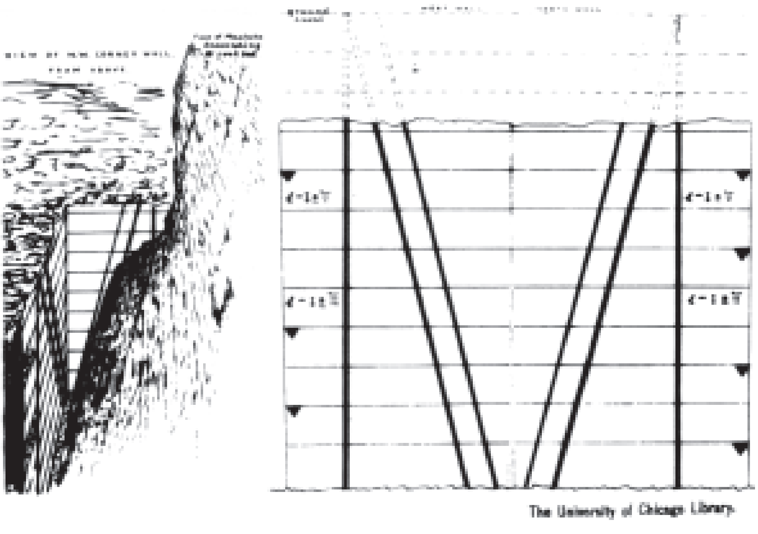
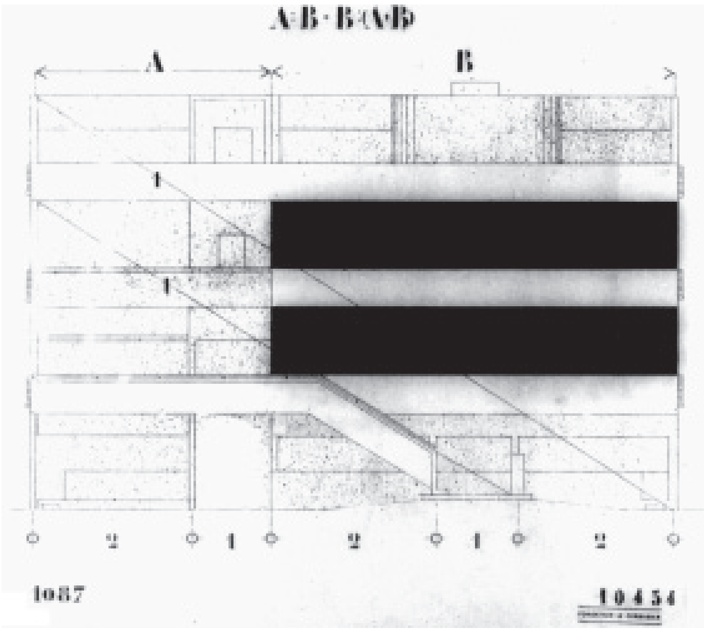


Figure 1

Figure 2



These two examples show that even when we have pictorial and written evidence, we must act with extreme caution in deciding upon the theoretical basis of an architectural structure. Then there is of course the ever present difference between theory and practice.[5] In my opinion, these examples of difficulties—and the others that will come up in the course of the problems—suggest how we should present proportions to students of architecture. We should present proportions as the theoretical thoughts of the architects of the past and we should insist that it is much more important to think about the theoretical basis, and the mathematical techniques involved, than to engage in an endless, and ultimately futile, search for "hidden" proportions.

We can now turn to the Case Studies themselves. Note that the presentation, and the questions asked, vary according to problem. The presentation depends upon whether I presenting the method as something that we definitely know something about or an example of speculation.

Case Study 1. The Great Pyramid of Egypt.

The Great Pyramid of Egypt was built about -2600, during the IVth dynasty (Figure 3). There have been at least eleven theories put forward to explain the theoretical basis of the shape. Five of the theories are listed below.

1. *Seked* theory: In ancient Egypt the basic linear unit of architectural measurement was a cubit of about 52.3 cm. Each cubit was divided into 7 hands of 4 fingers each. The opening quotation of this article cites a pyramid example from the Rhind Papyrus. From these examples we can ascertain that the *seked* of an inclined

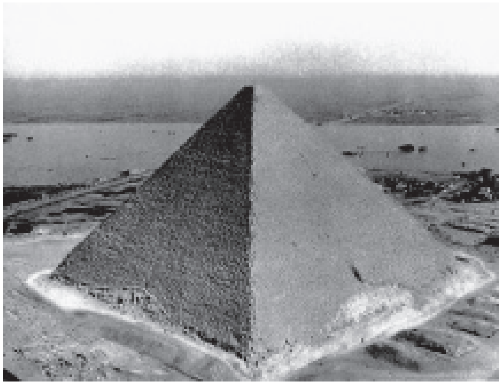


Figure 3

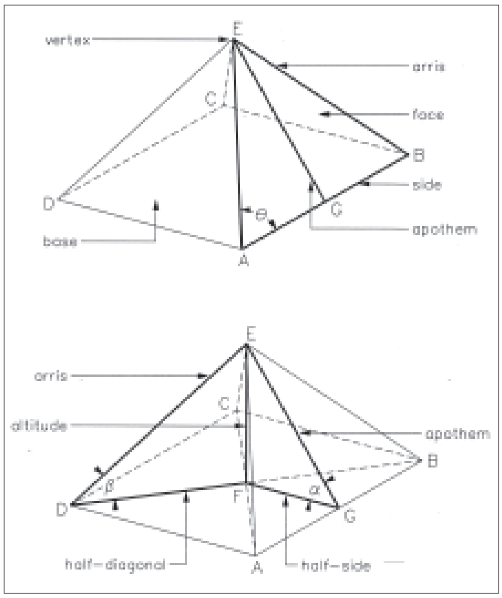


Figure 4

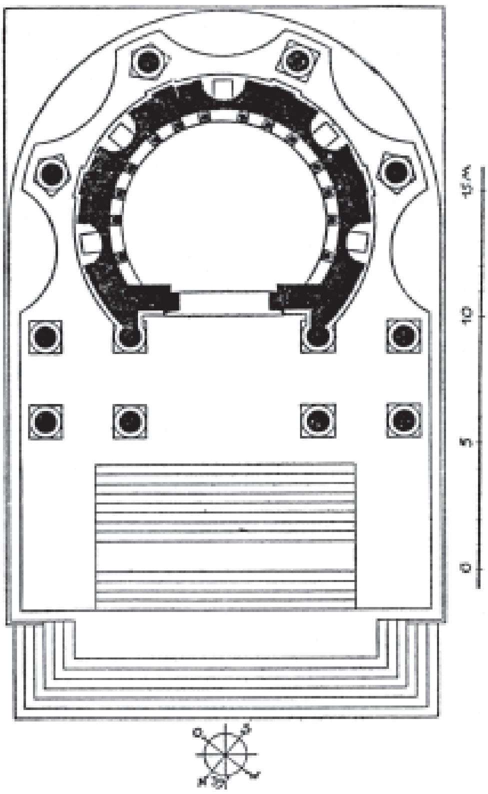


Figure 5

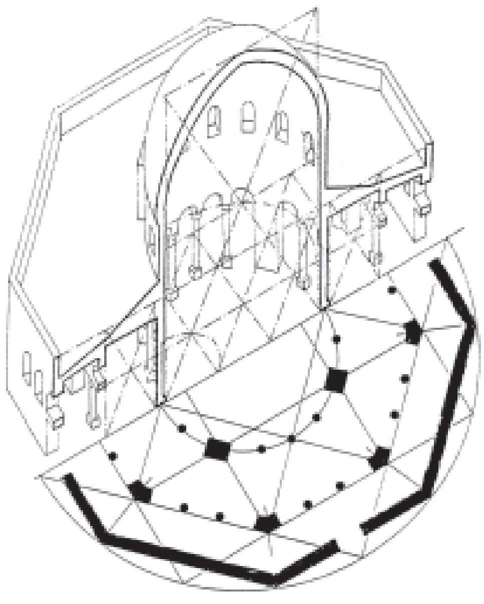


Figure 6

line was the horizontal run for a rise of 1 cubit. In the diagram shown in Figure 1 of the excavation of the mastaba at Medum the horizontal lines are one cubit apart. The *seked* theory of the Great Pyramid assumes that the Pyramid was built so that the *seked* was 5 hands and 2 fingers.

2. The arris had a rise over run ratio of 9:10.
3. The angle of inclination corresponds to that of a triangle inscribed in a regular heptagon.
4. "Area" theory: The Great Pyramid was built so that a square with sides equal to the height would have the same area as one of the faces.
5. "Pi" theory: The Great Pyramid was built so that the base would have the same perimeter as a circle whose radius was the height. For what well-known estimate for the value of pi does the angle given by the pi-theory exactly equal to that given by the *seked* theory?

i. For each of the following five theories find the angle of inclination (a) of the faces and compare it with the observed angle of inclination, which we take to be 51.84° (Figure 4) [Herz-Fischler 2000]
ii. For what well-known estimate for the value of p does the angle given by the "pi" theory exactly equal to that given by the *seked* theory?
iii. Which, if any, of the above the above theories would you accept?
iv. Would you accept more than one theory?
v. How many theories can you make up and which of the ones that you have made up would you accept?

Case Study 2. The Round Temple at Baalbeck.

The diagram shows the plan of the Roman temple at Baalbek (Lebanon, second century, Roman name Heliopolis) (Figure 5).[6] In discussing this temple, D. Robertson says, "The plan is based on an ingenious geometrical scheme which cannot here be explained" [Robertson 1929: 265].

Find the "ingenious geometrical scheme".

Case Study 3. The Dome of the Rock, Jerusalem.

The dome of the rock — Qubbat al-Sakhra — was built by the caliph Abd al-Malik and terminated in 72 A.H. (691), remarkably soon after Mohammed's flight from Mecca to Medina, the date which marks the beginning of the Moslem calendar. The diagram in Figure 6 shows a composite view of the dome [Choisy 1899, I]. The guiding principles behind the proportioning of the dome are unknown, although there has been much speculation.[7] Let us assume that because of space considerations it was the radius r1 of the outer circle that was of primary consideration.

i. Find the side of the octagon in terms of r1. Find the area of the octagon.
ii. Let r2 be the radius of the inner circle and h the height of the cylindrical part of the dome. Assume that the top of the dome is a half-sphere. Find the total volume of the cylindrical part of the dome.

Case Study 4. The Cathedral of Milan: Ad quadratum or ad triangulum?

A very enlightening indication of the methods used by the medieval architect is provided by the still extent records of a meeting of May 1, 1392 in connection with the Cathedral of Milan (Figure 7).[8] The minutes of the meeting record the following statement and then a question and answer:

> The solution of this dilemma was not in the province of an architect, and a mathematician ... was summoned.
>
> Question: Whether this church, not counting within the measurement the tower which is to be built, ought to rise according to the square (*ad quadratum*) or the triangle (*ad triangulum*).
>
> Answer: It was stated that it should rise up to a triangle or to the triangular figure, and not farther [Ackerman 1949: 90, 91].

To understand the meaning of this question and answer, it is necessary to consider the earlier history of the construction. The church had been started and its foundation laid with a width of 96 Milanese *braccia* (a measurement, like the Egyptian cubit, based on the length of the forearm, the Milanese *braccio* was about .595 meters). The original plans called for the elevation to be determined by a simple grid system with a total height of 90 *braccia*. However, there was a disagreement as to the aesthetic basis as well as the practical height of the cathedral. A second architect then suggested that the height be determined by an equilateral triangle (*ad triangulum*) with sides equal to the width of the cathedral and work proceeded to the 28 *braccia* level on this basis. Later, however, work was stopped and a third (!) architect wanted to use the square (*ad quadratum*) as the theoretical basis, i.e., the height should equal the width. The minutes of the meeting reflect this dispute.

i. Find the theoretical height *ad triangulum* in terms of *braccia* and meters.[9] Also find the difference in meters between the original proposed height and the *ad quadratum* height.

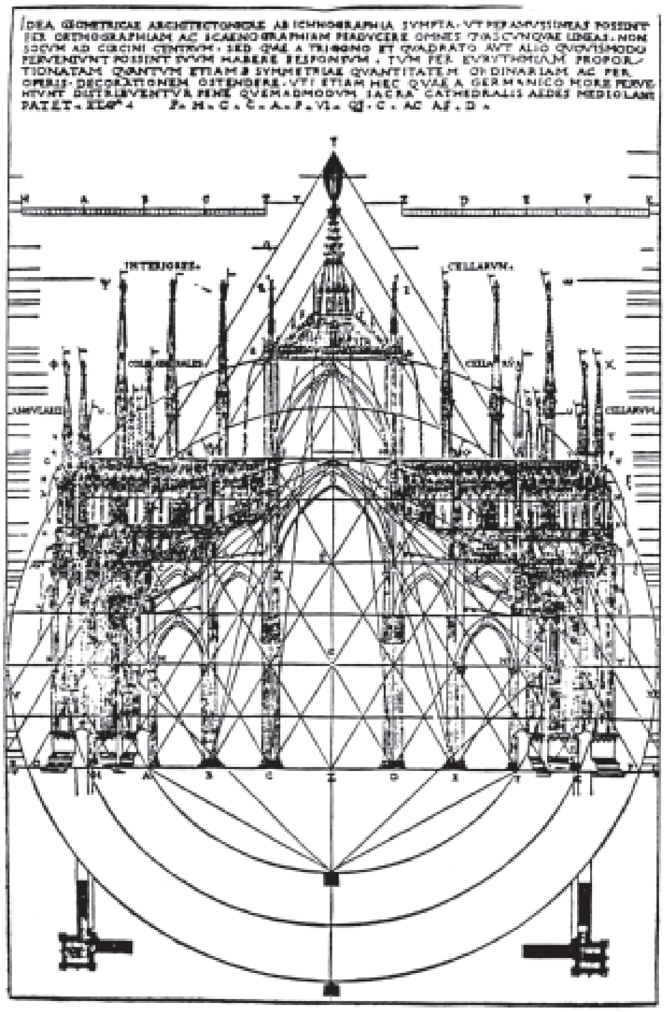
In fact the original “ad triangulum” plan was not followed for, as the answer says, the cathedral was to rise up to a triangle on the triangular figure”. What the council had really decided was to retain the other piers at 28 *braccia*, but to introduce a new set of 12 *braccia* (the width was 96 = 8 x 12 *braccia*). The height was then determined by two back-to-back isosceles right triangles of base 48 *braccia*, each sitting at the 28 *braccia* level. Thus in effect the equilateral triangle was completely ignored in the final version.

ii. Compare the final height with the various other plans (the actual cathedral was reduced by 1/2 *braccia* in execution but do not take this into account in your calculations).

Case Study 5. The Mathematical Methods of the Medieval Masons.

Mediaeval masons insisted that their whole craft was based on the “art and science of geometry.” It has been the purpose of this paper to reconstruct the character and content of the geometrical knowledge of mediaeval master masons from the few literary remains of the masons themselves. As reconstructed from these writings, this geometry scarcely

Figure 7



resembles either the classical geometry of Euclid and Archimedes, or the mediaeval treatise on *practica geometriae*. Mathematically speaking, it was simple in the extreme; once it is recognized that there was virtually no Euclidean-type reasoning involved, the way is cleared for understanding the kind of geometrical thinking which the masons did employ. This non-mathematical technique I have labelled constructive geometry, to indicate the masons' concern with the construction and manipulation of geometrical forms. It becomes evident that the "art of geometry" for mediaeval masons meant the ability to perceive design and building problems in terms of a few basic geometrical figures which could be manipulated through a series of carefully prescribed steps to produce the points, lines and curves needed for the solution of the problems. Since these problems ranged across the entire spectrum of the work of the masons — stereotomy, statics, proportion, architectural design and drawings — the search by modern scholars for the geometrical canons of mediaeval architecture is appropriate enough, so long as we keep clearly in mind the kind of geometry that was actually used by the masons. The nature of that geometry suggests that these canons, when recovered, will not be universal laws which will at last provide the key to mediaeval architecture; rather, they will be particular procedures used by particular master masons at particular times and places [Shelby 1972: 420].[10]

Figures 8 deals with the method of making templates used by a German master named Lorenz Lechler (1516) [Shelby 1972: 409 and 419]. It was based on a modular unit determined by the choir wall of the church: "Take the wall thickness of the choir, whether it be small or large, then draw squares through one another; therein you will find all templates, just as you will find them drawn in this book".

i. Explain the construction in Figure 8.

Figure 9 is taken from the *Geometria Deutsch* by Matthias Roriczer (1486) [Shelby 1971: 153].

ii. Explain the construction in Figure 9.

Figure 10 is taken from the "Vienna Sketchbook" [Shelby 1971: 153].

iii. Explain the construction in Figure 10.

Shelby givens the following method of finding the circumference of a circle, taken from Mattias Roriczer's *Geometria Deutsch* (1486), to show how medieval masons used step-by-step manipulation of their working tools to avoid mathematical calculations:

Whoever wishes to make a circular line straight, so that the straight line and the circular line are the same length: make three circles next to one another and divide [the diameter of] the first circle in seven equal parts, with the letters as shown h:a:b:c:d:e:f:g. As far as it is from h to a, set a point behind it [i.e., on the other side of h] and there put an i. Thereby, as far as it is from i to k., equally as long is the circular line of one of the three circles which stand next to each other as shown in the attached figure (Figure 11).

iv. Explain the text.

One place where more advanced techniques were used was in the requirement that the area of a spire be equal to the area of the ground plan, the basic figure of which was the square. Thus the mason was faced with a problem in transformational geometry, i.e., he had to construct one figure equal in area to another figure. Compare the following two methods of constructing a square equal in area to a given triangle (Figures 12 and 13).

In Figure 12, taken from an anonymous fifteenth century work, *De inquisicione capacitatis figurarum*, we start with triangle abc and then draw rectangle bced. The rectangle is then divided in half by line fg. Now extend bc to h with ch = cg = ge. Draw the semicircle on bh. Line ec extended determines k. It is claimed that kc is the side of the square whose area equals that of the original triangle.

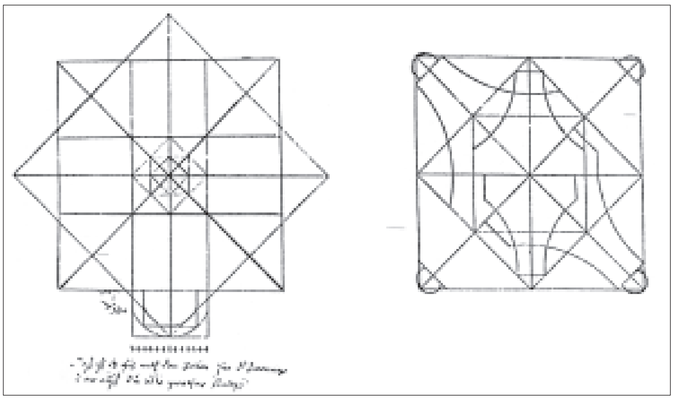


Figure 8

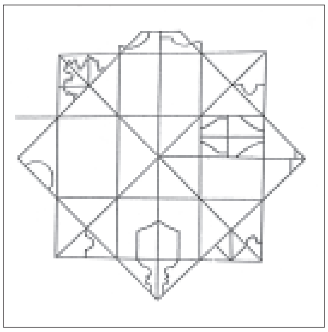


Figure 9

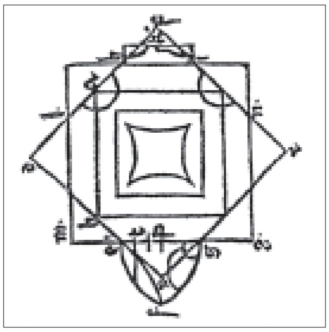


Figure 10

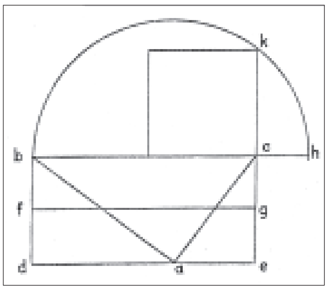


Figure 12

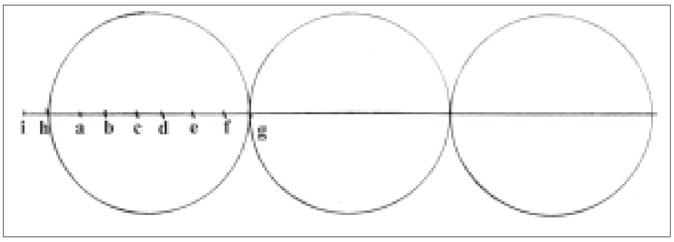


Figure 11

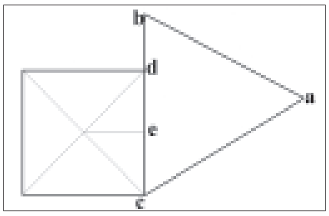


Figure 13

On the other hand, Figure 13, taken from Matthias Roriczer's *Geometria Deutsch* [Shelby 1972: 413], starts with an equilateral triangle abc and then divides bc into three parts at e and d. It is claimed that ec is the side of the desired triangle.

v. Are the methods exact?

Case Study 6. Francesco di Giorgio Martini's System.

Fifteenth century Italian architect Francesco di Giorgio Martini proposed the system shown in Figure 14, which essentially turns an incommensurable system involving the diagonal of a square into a modular system:

Let the side of the square ES represent the width of a room and then draw the "double square" of base EP. Next we draw the semicircle on EP and the diagonals QP and SM. It is then claimed that the disance RT is approximately 1/5 of SM and also approximately 1/7 of EP. It is now GS = 5TR that is taken as the height of the room. The module TR is also used to generate the length of the room [Hersey 1976: 137].

Check the statements concerning RT, SM and EP.

An example of a design based on the above system is shown in Figure 15.

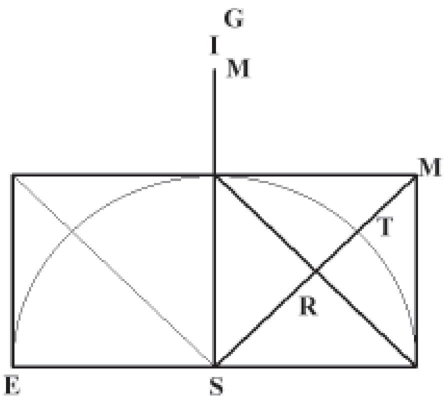


Figure 14

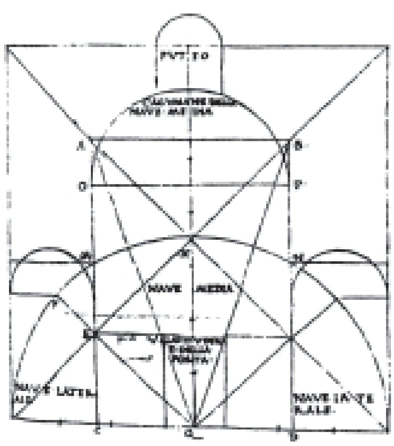


Figure 15

Case Study 7. Sebastiano Serlio's Apparent "Geometrical" Scheme.

> *To the minds of the men of the Renaissance musical consonances were the audible tests of a universal harmony which had a binding force for all the arts. This conviction was not only deeply rooted in theory, but also... translated into practice. It is true, that in trying to prove that a system of proportion has been deliberately applied by a painter, a sculptor or an artist, one is easily misled into finding those ratios which one sets out to find. In the scholar's hand dividers do not revolt. If we want to avoid the pitfalls of useless speculation we must look for unmistakeable guidance by the artists themselves* [Wittkower 1971: 126].

To illustrate the above statements, Wittkower, in his *Architectural Principles in the Age of Humanism*, gives us the following illustration taken from Serlio's *Libro primo d'architettura*. (Figure 16). It appears from the diagram that the height and the width of the door were determined by the equilateral triangle and diagonals of the square, but this is not the case; the lines were added afterwards. The width of the door, the height of the door and the side of the square are in the ratio 1:2:3 in typical Renaissance "harmonic proportion".

Where are the intersection points F and G of the line with respect to the corners of the door?

Case Study 8. Leonardo da Vinci's "Human Figure in a Circle"

Figure 17 is a reproduction taken from a manuscript in the Venice Academy of Leonardo's drawing (c. 1500) of the human body inscribed in a circle. The accompanying text reads:

> *The architect Vitruvius states in his work on architecture that the measurements of a man are arranged by Nature thus:*
> *"...that is that four fingers make one palm, and four palms make one foot, six palms make one cubit, four cubits make once a man's height, and four cubits make a pace, and twenty four palms make a man's height, and these measurements are in his buildings."*
>
> *If you set your legs so far apart as to take a fourteenth part from your height, and you open and raise your arms until you touch the line of the crown of the head with your middle fingers, you must know that the centre of the circle formed by the extremities of the outstretched limbs will be the navel, and the space between the legs will form an equilateral triangle.*
>
> *The span of a man's outstretched arms is equal to his height.*
>
> *From the beginning of the hair to the end of the bottom of the chin is the tenth part of a man's height; from the bottom of the chin to the crown of the*

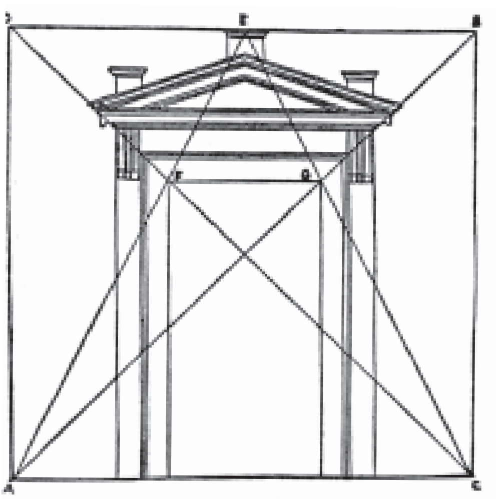


Figure 16

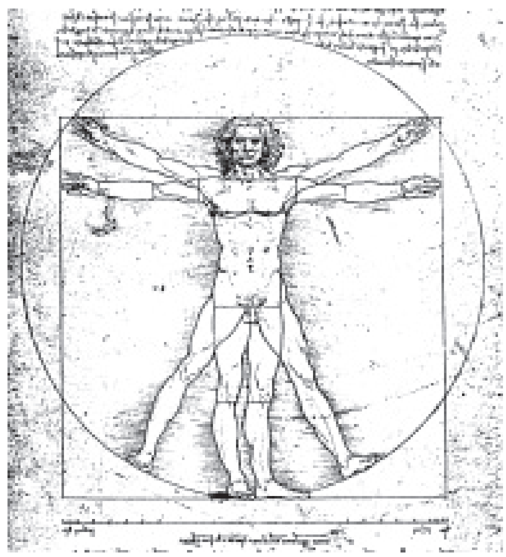


Figure 17

head is the eighth of the man's height; from the top of the breast to where the hair commences is the seventh part of the whole man; from the nipples to the crown of the head is a fourth part of the man. The maximum width of the shoulders is in itself the fourth part of a man; from the elbow to the tip of the middle finger is the fifth part; from this elbow to the end of the shoulder is the eighth part. The complete hand will be the tenth part. The penis begins at the centre of the man. The foot is the seventh part of the man. From the sole of the foot to just below the knee is the fourth part of the man. From below the knee to where the penis begins is the fourth part of the man.

The parts that find themselves between the chin and the nose and between the places where the hair and the eyebrows start each of itself compares with that of the ear, and is a third of the face [MacCurdy 1956, 1: 204, plate 13; Panofsky 1968].

i. Show the correspondence between the text and the diagram.
ii. Is the description consistent in the sense that if the position of any point is described verbally in two different ways, the point's geometrical location is the same in both cases? (Note, e.g., how the words "centre" and "penis" occur twice in different contexts.
iii. What is the distance of naval from the top of the head, according to this system? Does this agree with the following statement?

From the roots of the hair to the top of the breast is a sixth of a man's height; and this measure never varies. It is as far from the outside part of one shoulder to another as it is from the top of the breast to the navel, and this goes four times into the distance from the sole of the foot to where the bottom of the nose begins. The arm, from where it separates itself from the shoulder in front, goes six times into the space between the two extremities of the shoulders and three times into a man's head and four into the length of the foot and three into the hand whether on the inside or the outside [MacCurdy 1956, 1: 199]

There are several things that should be remarked. First of all, Leonardo is basing his system on the statements of Virtruvius, as mentioned in the introduction, i.e., simple proportions, circle, square.

Secondly, although Leonardo's "man in a circle" is the best-known example, it is far from being the only Renaissance attempt at making a drawing based on the statement of Vitruvius.[11] The following examples that connect the "human in a circle" with letters of the alphabet are taken from Geofroy Tory's *Champ Fleury* (1529) [Tory 1970] (Fig. 18).

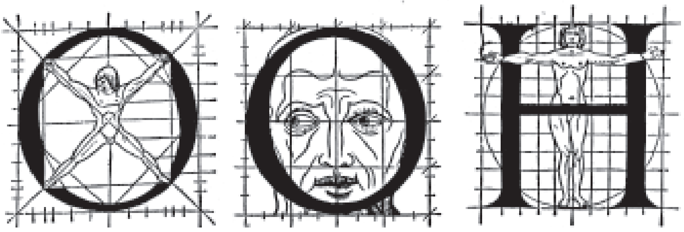


Figure 18

Case Study 9. Viollet de Duc's "Analysis" of the Parthenon.

The French architect-historian-reconstructor (e.g., the towers on the Cathedral of Clermont-Ferrand, Carcassone) was one of many people who tried to analyze the Parthenon. His solution involved the following triangle (Figure 19) [Viollet-le-Duc 1863: 399].

Start with a pyramid such that a section parallel to one of the edges of the square base is an equilateral triangle. Now consider the triangle obtained when we take a section along the diagonal. This triangle is the one that "fits" the Parthenon. But there is absolutely no historical basis to either Viollet le Duc's solution or any of the other exotic ones that have been suggested.[12]

Find the base angle of the latter triangle obtained by taking a section along the diagonal

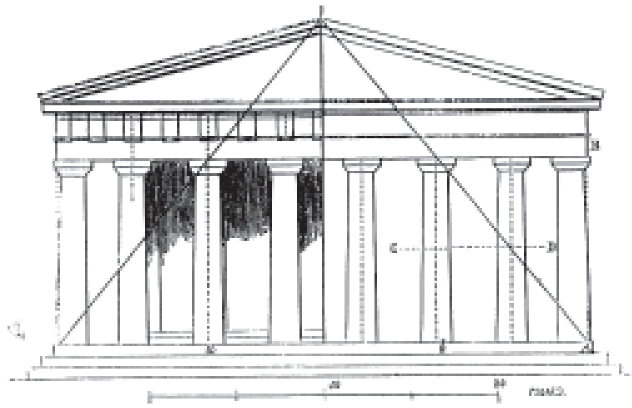


Figure 19

Case Study 10. The Place of the "Right Angle".

The place of the right angle was the dominating theme in Le Corbusier's use of proportions in architectural projects until 1928. We can see an example of its use in the villa at Garches (1927), both in the "official" version of Figure 20 that appears in Le Corbusier's *Oeuvre Complète* [Stonorov and Boesiger 1937:144] [13] and the preliminary sketch Figure 21.[14]

Figure 22 shows the facade of the type C1 houses at Stuttgart (1927) [Le Corbusier 1929: 12].[15] According to *The Modulor*, Le Corbusier discovered the principle of the place of the right angle while analysing Michelangelo's Capital in Rome [Le Corbusier1968:26]. This however would appear to be another late addition to Le Corbusier's view of his life and work.[16]

It should be noted that nowhere does Le Corbusier ever say what exactly the "place of the right angle" is supposed to do from a mathematical viewpoint. Le Corbusier himself, as he states, understood little of the strictly mathematical details of what he was doing and he even wrote in 1925, "I studied mathematics, but it did not help me later on. Perhaps however it helped form my intellect" [Le Corbusier 1968: 29; Herz-Fischler 1984: note 9]. What is so interesting about Le Corbusier is his interest in using mathematics and science despite his educational handicap.

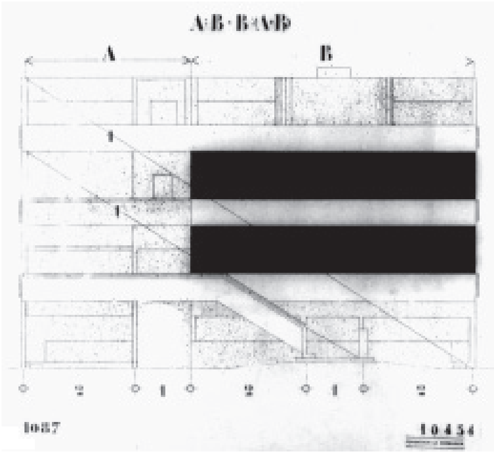


Figure 20

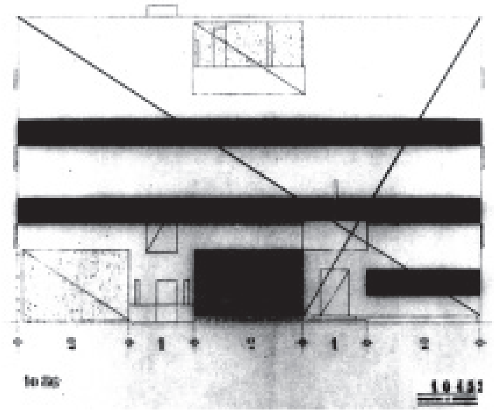


Figure 21

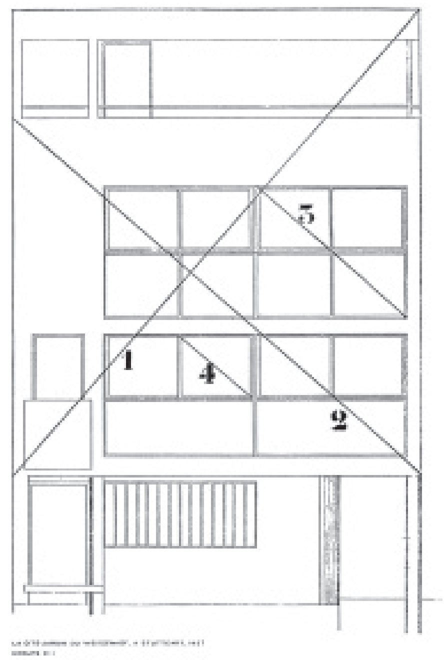


Figure 23

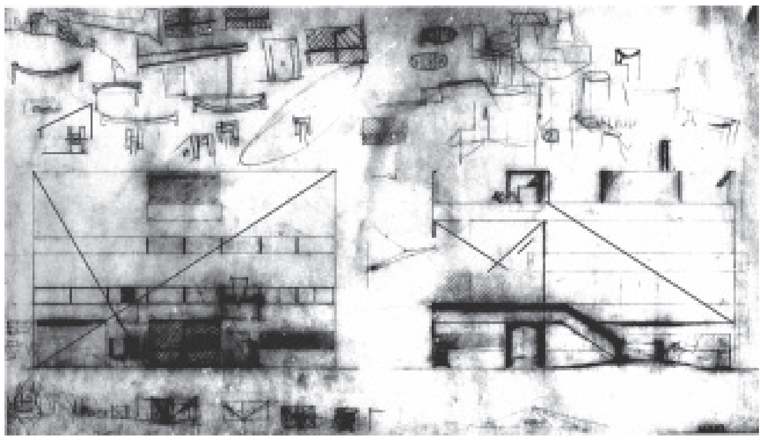


Figure 22

From a technical point of view, the “place of the right angle” may be explained by means of the diagram in Figure 23.

i. Start with an arbitrary rectangle ABCD. Draw the diagonal AC. Now draw BE perpendicular to AC; the intersection point is “the place of the right angle.” If we now form rectangle BCEF, then this rectangle is similar to the original rectangle ABCD. Furthermore, the same is true if we repeat the same procedure at point C, so that the rectangle BHGC is now exterior to ABCE, or if we start an arbitrary point, such as N on the diagram, and again obtain a similar rectangle.
Show that these claims are correct.

A word of caution is in order concerning the analysis of Le Corbusier’s works involving the “place of the right angle”. Namely, it is not always possible to tell where he started. Sometimes it is not clear how certain lines were drawn (see the opening quote with Case Study 11 below), and sometimes it appears that several points may have been the starting point. An example of this is Maisons La Roche-Jeanneret (1923) [Stonorov and Boesiger 1937: 68; *Le Corbusier Archives*, 1: nos. 15183, 15232, 15255] (Figure 24).

ii. “Analyze” Maisons La Roche-Jeanneret. How was point A5 chosen?

If we now return to drawing 1087 shown in Figure 2 and compare it with the preliminary sketch 1086 shown in Figure 20 we notice that something has been added, namely the “golden number” relationship A:B = B:(A+B). By checking the Atelier Record Book at the Foundation Le Corbusier I learned that this drawing was made at least 1-1/2 years after the plans for Garches were drawn and approximately a year after the building was completed. Interestingly in his earlier writings Le Corbusier had shown himself to be strongly against the use of the “golden number” [Herz-Fischler 1979]. Then under the influence of Ghyka’s *Esthétique des proportions dans la nature et dans les arts*—which appeared sometime in mid-1927 and thus at a point in time when the building was finished or almost so—Le Corbusier completely changed his attitude to the “golden number”. Not only did he become one of the strongest advocates of the use of the “golden number”, he changed his drawing of Garches so as to make readers believe that he had used it as a design principle [Herz-Fischler 1984]! Ghyka’s books caused not a few people—I being no exception—to believe in the “golden number” both as a historical reality and as the basis of architectural design.

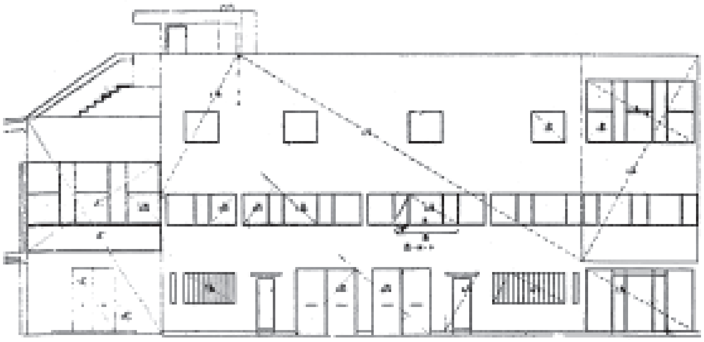


Figure 24

Case Study 11. Proportion in Le Corbusier's Paintings

> *L'exégète non-averti pourra s'évertuer sans succès à reconnaitre en ces oeuvres des tracés...il n'y arrivera pas ou il tombera dans l'arbitraire* [Le Corbusier 1949: 214].

In addition to being an architect, Le Corbusier was a painter. Together with Amedée Ozenfant he founded the so-called "purist" school of painting in reaction to certain tendencies in Cubism. It was during this period that Charles–Edouard Jeanneret adopted the name Le Corbusier, "the crow".[17]

The purist paintings of Jeanneret and Ozenfant were based on a well-defined system, which, however, varied somewhat from one period of time to another. The basic canvas size was 81 cm x 100 cm. Inside this canvas two equilateral triangles, facing in opposite directions, were drawn. The two intersections points of the triangles determined two "places of the right angle" (see Case Study 10), which in turn determined the vertices of two right triangles with a second vertex coinciding with a vertex of the equilateral triangle. The setup is shown in the diagram in Figure 25.[18] Figure 26 shows *Nature morte à la pile d'assiettes* (1920), perhaps Le Corbusier's most famous painting [Le Corbusier 1968: 213, fig. 92]. Underneath we have his analysis, featuring the two equilateral triangles and the two "places of the right angle".

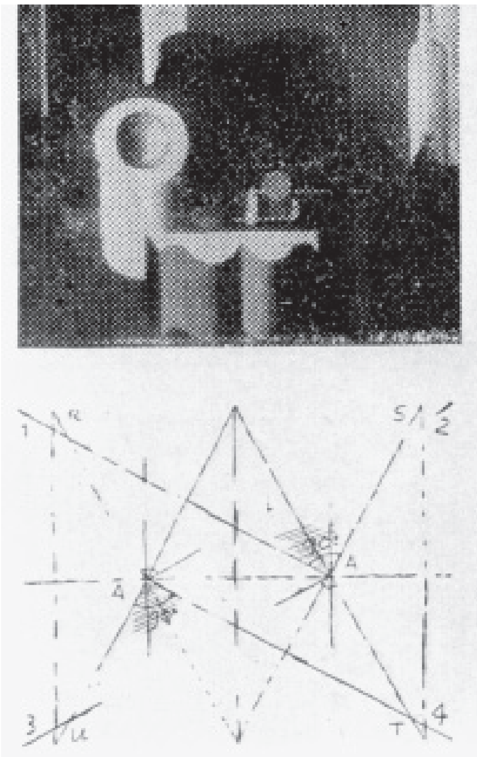


Figure 26

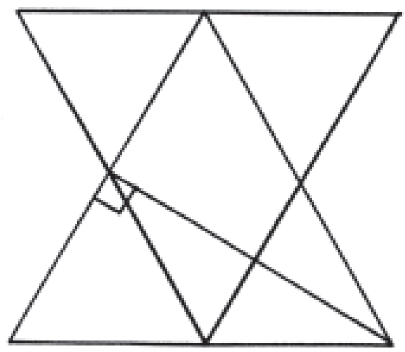


Figure 25

Note that there is no “golden number” proportion in the analysis, indeed Le Corbusier’s own writings show that he was strongly against the use of the golden number at that time (see Case Study 10).

Consider the upper right triangle and assume that, as indicated at the numeral “5” that its right leg starts at the right vertical line. The other, longer, leg crosses the inverted equilateral triangle to the left of the left vertical line. How far to the left of the left vertical line does the left vertex of the upper right triangle lie?

Case Study 12. Le Corbusier’s “Modulor”.

> *Are there not two kinds of arithmetic, that of the people and that of the Philosophers?* [Plato, *Philebus* 56D]
>
> *Except in the smallest elements (i.e., nails, hinges) disciplines of absolute standardization are unnecessary. Mathematically derived proportion is a confidence trick* [Smithson and Smithson 1970: 94].

The system of proportion known as the Modulor was first presented in the 1948 book of the same name. Although Le Corbusier used it in his later works [Le Corbusier 1968], and although it is often talked about, there are in fact few architects who actually used this system. One example is Sert’s Eastwood Project on Roosevelt Island, New York City, published in the August 1976 issue of *Architectural Record*.

Stripped of all its romantic elements, the Modulor system is very straightforward: basic heights a, 2a (113 cm for the “red” series and 226 cm for the “blue” series) are chosen and one then simply multiplies these heights by increasing and decreasing powers of the “golden number” to obtain the values in the series. (The “golden number” = $(1+\sqrt{5})/2$). It is often denoted by the Greek letter phi (ϕ), but I prefer to denote it by the letter G in order to avoid entering into the controversy of the applications of the “golden number”.)
Thus: $aG^{-2}....aG^{-1}.... a .... aG .... aG^{2}....$
(likewise for 2a). The actual values are tabulated in *The Modulor* [Le Corbusier 1949: 82, fig. 3].

Now what makes these numbers into a “Modulor system” is the fact that if we add any two numbers in the sequence we obtain the next term.. This is called the Fibonacci property.

i. Show that $G^{n+2} = G^{n+1} + G^{n}$ and that $1/G^{n+2} + 1/G^{n+1} = 1/G^{n}$ for all positive integers n (hint: start with the fact that G satisfies all $x^2 = x+1$).

Le Corbusier, however, was not satisfied in presenting the Modulor system in just this straightforward form. It was his desire to connect the “golden number”-based sequence with the “place of the right angle” (see Case Study 10). The solution to this may be explained in Figure 27 as follows:

On the lines CD and DE of length h construct two squares. Divide CD at F and DE at H according to the “golden number”, i.e., $DH = G^{-1}h$, etc. Now draw the right angle in the “double square” with vertex at D. The points A_{-1} and A_0 are now determined, as is the line $A_{-1}A_0$, which meets CE extended at S. By drawing another right angle at A_0 we determine G_1. A right angle at G_1 in turn determines A_1, and so on. We can also work upwards starting at A_{-1}. The point B_i is just the point on the line CS that is opposite A_i. On the cover of the Modulor, the arcs have their ends at the B_i.

ii. Show that B_1 coincides with E.

iii. Show that G_1 is always the golden number point of line $\overline{B_{i-1}B_i}$.

iv. Show that $\overline{B_{-1}B_0} - h$ and that the $\overline{B_iB_{i+1}}$ distances are just the terms of the Modulor series.
v. What happened to C? What did the squares have to do with all this?
vi. Find B_0S.
vii. What is the significance of the constructions to the right of the Modulor Man shown in Figure 28?

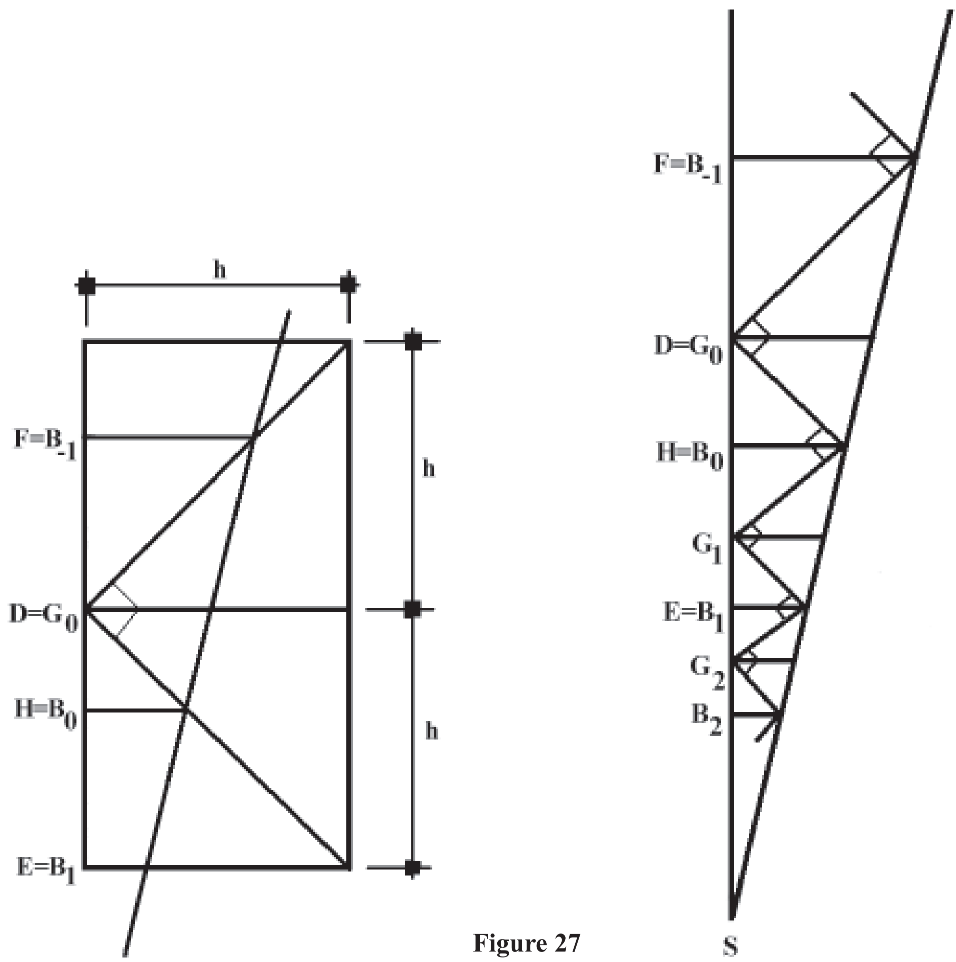


Figure 27

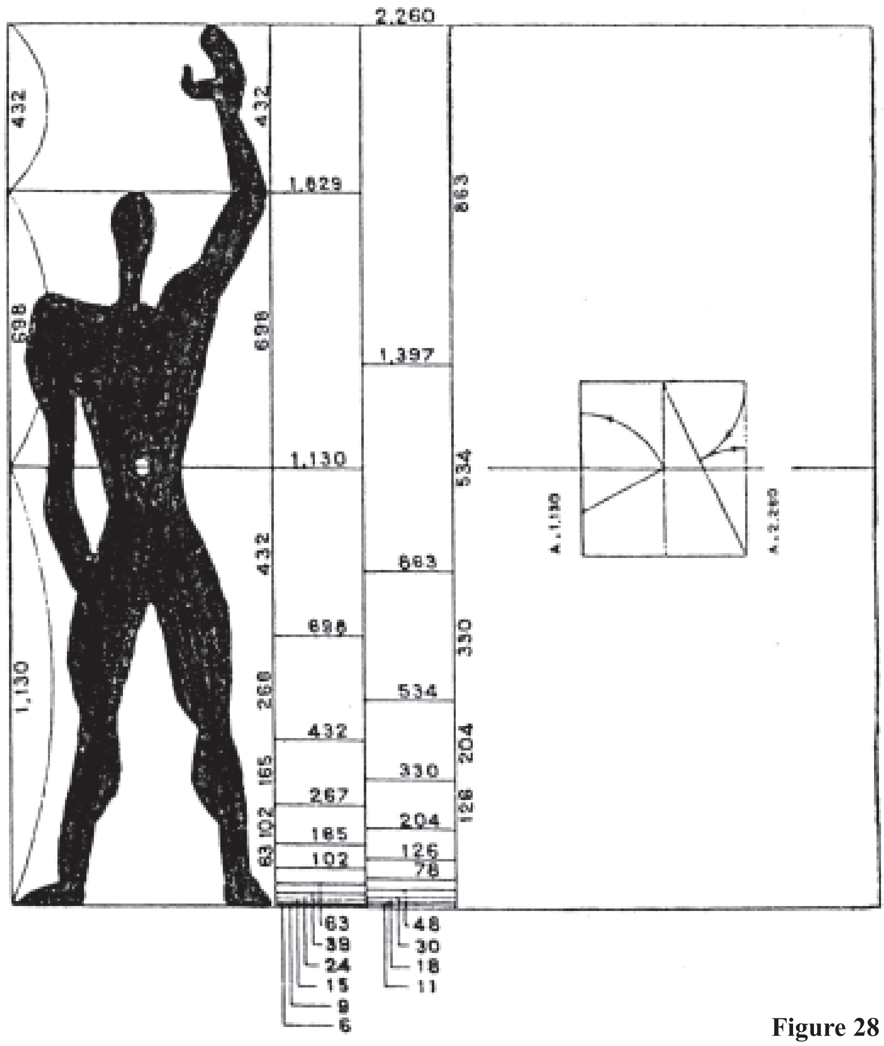


Figure 28

Notes:

1. The Rhind Papyrus is from the Fifteenth Dynasty (c. - 1800), but is known to be based on a Twelfth Dynasty text. For details of the pyramid problems, as well as a discussion of the archaeological, historical and philosophical aspects, related to the problem of determining how the Great Pyramid was designed, see [Herz-Fischler 2000].

2. The school year 1983-1984 was the last time that I taught the course. The new director believed that poetry was more important than mathematics and that I should limit the mathematical level to constructing models of dodecahedra etc. I informed the chairman of the Department of Mathematics that in good conscience I could no longer teach the course. After another two years the school dropped the mathematics course and most of the other "scientific" courses from the

curriculum. As the subtitle of the book suggests, I tried to teach the students how to analyse architectural objects and situations with a view to actually obtaining numerical values for the various dimensions involved. I completely eschewed such abstract topics as regular polyhedra, groups etc.

3. For a complete list of my articles and books, the reader may consult my web page, http://mathstat.carleton.ca/~rhfischl/golden_number/publications.txt.

4. The first part of the book dealt with the techniques and tools of solving real problems with an emphasis on a decomposition of the problems into small steps, each of which could solved by simple formulae (Pythagorean theorem, etc.). This is the "An Algorithmic Approach" referred to in the title of the book. The second part of the book was the problem section. After the first chapter on proportions came planar problems, spatial problems, true shape problems, conics, curvature and optimization. As examples, problem II.8 dealt with the overhang required to provide shade when the sun was at a certain angle; problem II. 13 dealt with difficulties (placements of columns, projectors, etc.) that I encountered in one of the Carleton School of Architecture's classrooms; problem II.28 involved the analysis of a housing project in Cluj, Rumania; II.33 with the allowable location of seats in the physics lecture halls at the University of Colorado; III.11 and III.12 were based on the Wright's Guggenheim Museum and dealt with spiral ramps, etc.

5. [Howard and Longair 1982] demonstrates in a scientific and forceful way the difference between theory and practice.

6. This example was kindly pointed out to me by Helmut Schade. Some color plates of Baalbek may be found in [Laroche 1979: 142].

7. For an example see [Creswell 1969, I, 1: 73].

8. The records are of great interest to the modern architect for they show how little the difficulties and relationships with clients have changed in 600 years! We also remark that the pitfalls of the committee system are nothing new. To top it all off, the committee hurled invectives at the poor architect: "He served the building badly, and in the end he gave great loss and damage to the building by reason of his own malfeasance." See [Ackerman 1949: 96, footnote 42]. See also [Frankl 1945]; [Rooseval 1944]; and a very interesting recent book, [Padovan 1999], especially p. 181.

9. For the mathematical computation at that time, see [Frankel 1945: 53 and Appendix]. The figure obtained was rounded off to 84 *braccia*, which gave 6 times a unit of 14 braccia.

10. For related material see: [Branner 1957]; [Koop and Jones 1933]; [Shelby 1971]; [Shelby 1972]; [Shelby 1961]; [Shelby 1965]. Note: The papers by Shelby are an excellent source of additional references.

11. Other examples of a "man in a circle" are given in [Wittkower 1971: 14, Plates 1-4].

12. To compare these fanciful flights of imagination with ancient techniques, see [Dinsmoor 1923-I], [Dinsmore 1923-II] and [Dinsmoor 1950].

13. The number 1086 in Figure 21 corresponds to the number in the Atelier Record Book which is preserved in the Fondation Le Corbusier in Paris, and it is this number which enables us to date the drawing. This particular reproduction is taken directly from the original microfilm in the Fondation Le Corbusier. The number 10453 was added by the Fondation Le Corbusier and it is under this number that the drawing appears in Le Corbusier Archives. Unfortunately these archive

numbers do not always follow a chronological order, even when these could have been determined by means of the atelier numbers.

14. Figure 22 is the only preserved preliminary sketch by Le Corbusier for Garches that shows any "regulating lines". This particular reproduction is taken directly from the original microfilm in the Fondation Le Corbusier, but some of the lines have been electronically enhanced for reproduction purposes. In the Archives reproduction not all of the lines are visible. Note in particular the triangles inside rectangles on the bottom left.

15. For full details, citations and references to the various drawings with "regulating lines", see [Herz-Fischler 1984].

16. It should also be noted that Le Corbusier himself gives other versions of the discovery of the "principle", see [Herz-Fischler 1984: note 9]. For recent discussions of Le Corbusier's introduction to the method, [Vaisse 1997] and [Padovan 1999: 28, 318]. There does not seem to be any evidence that Michelangelo or anybody else used the "place of the right angle". In any case, this concept remained an important one for Le Corbusier and in 1955 he even wrote a poem about it, *Poème de l'angle droit*.

17. On Le Corbusier's paintings, see [Wohl 1971], which contains many plates of Le Corbusier's paintings. For Ozenfant's version of the name "Le Corbusier", which differs quite markedly from that of Le Corbusier, see [Ozenfant 1968: 113].

18. For details on the system, see [Fischler 1979].

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